CIRCULANTS (Extract)

Alun Wyn-jones

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Notes and References

[AbS] "Handbook of Mathematical Functions", M. Abramowitz & I.A. Stegun (eds.); Dover Publications, Inc., 1970, (9^{th} printing).

[AD] "Zero-sum sets of prescribed size" by Noga Alon & Moshe Dubiner, pub.at School of Mathematical Sciences, Tel Aviv University, 2006. www.math.tau.ac.il/ nogaa/PDFS/egz1.pdf

[BaP] "Boolean Circulants, Groups, and Relation Algebras" by Chris Brink & Jan Pretorius; American Mathematical Monthly, **99**, 1992, pp 146-152.

[Bass] "The Dirichlet Unit Theorem, Induced Characters, and Whitehead Groups of Finite Groups." by Hyman Bass. Topology, **4**, 1966. pp 391-410.

[Dav] "Circulant Matrices" by Philip J. Davies; John Wiley & Sons, 1979. Available from University Microfilms International, Ann Arbor, Michigan. This is the only other extant book on circulants. It is excellent reading and explains clearly all the basic properties of circulants and much more besides. It has an intriguing chapter on the application of circulants to polygonal geometry.

[Dav1] ibid. §2.5.

[Dav2] ibid. §2.5. Davies derives the basic properties of the Fourier matrix. See also [Fla] below.

[Dav3] ibid. §5.3.

[Dav4] ibid. §5.6.

[EDM] "Encyclopedic Dictionary of Mathematics" by the Mathematical Society of Japan, editors Shôkichi Iyanaga & Yukiyosi Kawada; The MIT Press, 1980, §107G, p350.

[Edw] "Fermat's Last Theorem, a Genetic Introduction to Algebraic Number Theory" by H. M. Edwards; Springer-Verlag, 1977.

[Edw1] ibid. §4.3.

[FB] "Einleitung in die Theorie der binären Formen." by Francois Faà di Bruno; Leipzig (1881).

This is the reference given by Ore in his paper. See [Ore]

[FG] "The Prime Factors of Wendt's Binomial Circulant Determinant" by G. Fee & A. Granville; Mathematics of Computation **57**, 1991, pp 839-848.

[FLA] "Introduction to Number Theory" by Daniel E. Flath; John Wiley & Sons, 1989. The definition of the discrete Fourier transform agrees with that in §6.5 of Flath's book which has more information on the F matrix. The other common definition of the Fourier matrix is $(1/\sqrt{N})\sum_j a_j\zeta^{-ij}$.

[FT] "Algebraic Number Theory" by A. Fröhlich & M.J. Taylor; Cambridge University Press, 1991. §1.44.

[Gauss] "Disquisitiones Arithmeticae" by C.F. Gauss; published in Leipzig, 1801. English translation by A.A. Clarke, Yale University Press, 1966. See Art. 24.

[Guy] "Unsolved Problems." by R. Guy, (ed.); American Mathematical Monthly, **100**, 1993, pp 287-289.

[Ham] "The Friendship Theorem and Love Problem" by J.M. Hammersley; Surveys in Combinatorics (9^{th} British Combinatorial Conference) E. Keith Lloyd(ed.) available in London Mathematical Society Lecture Note Series **82**, Cambridge University Press, 1983, pp 31-54.

[HaW] “Introduction to Theory of Numbers” by G.H. Hardy & E.M. Wright; Oxford University Press, 1960, (4^{th} ed.).

[HaW1] ibid. Theorems 67 and 272.

[Joh] “Presentation of Groups” by D.L. Johnson; London Mathematical Society, Series 22, Cambridge University Press, 1976, Chapter 16.

[Kap] “Commutative Rings” by I. Kaplansky; University of Chicago Press, 1974 (revised edition).

[Kap1] ibid. To see how such a field can constructed from arbitrary commutative rings see §1-4. See also [Lang].

[Kap2] ibid., (Hilbert Basis Theorem) Thm.69

[Kap3] ibid., (Dedekind domains) Thm.98

[Kar1] “Unit Groups of Classical Rings” by G. Karpilovsky; Oxford University Press, 1988.

[Kar2] ibid., §8.9.31.

[Kar3] ibid., §2.2.10.

[Kar4] ibid., §2.2.12

[Kar5] ibid., Attributed to Kaplansky in §8.9.34.

[KW] “Polynomial Equations and Circulant Matrices” by D. Kalman, J.E. White; American Mathematical Monthly, **108**, 2001, pp 821-840.

[Lam] “On rational circulants satisfying $A^2 = dI + \lambda J$.” Linear Algebra and Its Applications **12** 1975, pp 139-150.

[Lang] “Algebraic Number Theory” by Serge Lang; Springer-Verlag, 1986.

[Lang1] ibid., Chapter IV, Theorems 3 and 5. The proof in Lang’s book is quite general but requires reading and understanding most of the previous sections in the book. The reader may prefer instead to refer to [WAS5] which has a self-contained proof for the case n = prime.

[Lang2] “Cyclotomic Fields I and II” by Serge Lang; Springer-Verlag, 1990 (combined edition). §6.1

[LWW] “The Combinatorics of a Three-Line Circulant Determinant” by N.A. Loehr, G.S. Warrington, H.S. Wilf; Israel Journal of Mathematics, **143**, 2004.

[Muir] “A Treatise on the Theory of Determinants”, Sir Thomas Muir & W. Metzler; Longmans, Green, (New York), 1933. This book is an American edition of an earlier work by Muir. It was quite obviously intended as a general textbook on determinants but has a long section on circulants. Muir was active in the development of circulants in the 19^{th} century, and it seems he took the opportunity in this book to (briefly!) summarize the main results known at the time since much of the section would be of interest only to contemporary researchers in circulant determinants.

Please note the typographical error in Exercise 2 on page 443: all negative signs should be changed to plus signs.

[Muir1] ibid., Art.499. Although only the case $N = 4$ is proven, the general method is clear from the context.

[Muir2] ibid. There is a formula derived in Art.491, page 458 which is an apparent generalization to compound n of the formula derived herein at Proposition 10.7.3 which is proved for n prime only. However, the method by which Muir obtains the formula is valid only when n is prime. Indeed, should the reader wish to continue to the subsequent section, the formula should be taken as the definition of the variable S.

[Muir3] ibid. See §503, page 476 where the proposition is demonstrated for $n = 4, m = 3, N = 12$.

[Ore] "Some Studies on Cyclic Determinants" by Oystein Ore; Duke Mathematical Journal, **18**, 1951, pp. 343-354.

[Pan] "On a Congruence Modulo a Prime" by Hao Pan, American Mathematical Monthly, **113**, 2006, pp 652-654.

[Pas1] "The Algebraic Structure of Group Rings" by D.S. Passman; John Wiley & Sons, 1977. This is the standard reference on group rings.

[Pas2] ibid., Chapter 14, Theorem 1.2, attributed by Passman to Perlis & Walker. See also the preceding Lemma 1.1.

[PFTV] "Numerical Recipies. The Art of Scientific Computing" by Wm. H, Press, Brian P. Flannery, Saul A. Teukolsky, Wm. T. Vetterling; Cambridge University Press, London, 1986.

[PP] Preprints and references for various results in this text can be found at www.circulants.org.

[Rib1] "Fermat's Last Theorem for Amateurs" by Paulo Ribenboim; Springer-Verlag, New York, 1999.

[Rib2] "The Little Book of Big Primes" by Paulo Ribenboim; Springer-Verlag, New York, 1991. p15

[Rot] "The Theory of Groups" by Joseph J. Rotman. Allyn & Bacon, 1965.

[Seg] "Units in Integral Group Rings" by Sudarshan K. Seghal. Longman Scientific & Technical, 1993.

[Was] "Introduction to Cyclotomic Fields" by L.C. Washington; Springer-Verlag, 1982.

[Was1] ibid., Appendix §1 and Chapter 12.

[Was2] ibid., §2.4.

[Was3] ibid., Lemma 8.1 and Theorem 8.1. See also the more difficult Theorem 8.3 for general N.

[Was4] ibid., §5.36

[Was5] ibid., §1.2. This proves the integral elements of $\mathbb{Q}(\zeta_p)$ to be $\mathbb{Z}(\zeta_p)$ for p prime.

[Was6] ibid., §5.36

[Weil] "Basic Number Theory" by André Weil, Springer-Verlag, New York, 1973.

[Wyn] "Circulants", A. Wyn-jones. Manuscript for the complete text available at www.circulants.org.